

Global Linear Complexity Analysis of Filter Keystream Generators

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Abstract

An efficient algorithm for computing lower bounds on the global linear complexity of nonlinearly filtered PN-sequences is presented. The technique here developed is based exclusively on the realization of bit wise logic operations, which makes it appropriate for both software simulation and hardware implementation. The present algorithm can be applied to any arbitrary nonlinear function with a unique term of maximum order. Thus, the extent of its application for different types of filter generators is quite broad. Furthermore, emphasis is on the large lower bounds obtained that confirm the exponential growth of the global linear complexity for the class of nonlinearly filtered sequences.

1 Introduction

Many procedures in modern communication systems require binary sequences which appear to be random but, in fact, have been generated in a deterministic way. They are the so-called pseudorandom sequences. In cryptographic applications the sequence obtained in such a way is referred to as the keystream. To provide secure encryption the keystream must verify several properties of cryptographic nature such as: long periods, balanced statistics, mth-order correlation immunity, distance to linear functions, avalanche criterion... (for a more detailed survey see [9]). In addition a keystream generator has to be unpredictable: that is, given a portion of the output sequence, a cryptanalyst should be unable to predict other bits forward or backward. A widely accepted measure of the unpredictability of a sequence is the linear complexity defined as the shortest linear recursion over $\text{GF}(2)$ satisfied by such a sequence.

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One of the most commonly used keystream generators is obtained by applying a nonlinear function to the stages of a maximal-length Linear Feedback Shift Register (LFSR). This type of generator is called ‘filter generator’. The linear complexity of the resulting keystream can be computed in two different ways:

- 1.- Analysing the digits of the output sequence by means of the Berlekamp-Massey LFSR synthesis algorithm [6].
- 2.- Studying the nonlinear function applied to the LFSR’s stages.

Local linear complexity and global linear complexity are obtained in each case respectively. The global linear complexity of the filter generators depends exclusively on the particular form of the filter and the LFSR’s minimal polynomial. Generally speaking, there is no systematic method to predict the resulting global linear complexity. This is the reason why in the open literature statements like ‘it is extremely difficult to lowerbound (or guarantee) the linear complexity of the sequences produced by nonlinearly filtering the state of an LFSR’ [8, pp. 57] can be found. Nevertheless, some authors have faced this problem and several references can be quoted. Apart from the works of Groth [2] and Key [3], Kumar and Scholtz [5] derived a general lower bound for the class of bent sequences, although the LFSR’s length is restricted to be a multiple of 4. Rueppel [8] established his *root presence test* for the product of distinct phases of a PN-sequence, which is based on the computation of determinants in a finite field. One of the most recent works on this subject, [7], has focussed on the use of the Discrete Fourier Transform Technique to analyse the global linear complexity. Most of the above mentioned works impose rather restrictive conditions on the LFSR’s length, the order of the nonlinear function or the particular form of the applied function.

Based on the works [8] and [1], a new algorithm (the so-called LB-algorithm) is proposed for the computation of lower bounds on the global linear complexity. This algorithm can be applied to any arbitrary nonlinear filter with a unique term of maximum order. In fact, no restrictions are imposed on the LFSR’s stages, the particular form of the filter or the LFSR’s minimal polynomial. On the other hand, the most important feature of the LB-algorithm is that it is based exclusively on the realization of bit wise logic operations (OR, AND and XOR), which makes it rather adequate to either software simulation or hardware implementation.

As the algorithm INPUTS are L (LFSR’s length) and k (order of the function), then the lower bound obtained is valid for any k th-order function with a unique term of maximum order and for any LFSR of length L .

2 Review of the Root Presence Test and new Definitions

Some fundamental concepts and notation which are used in this work can be introduced as follows.

S is the output sequence of an LFSR whose minimal polynomial $m_s(x) \in GF(2)[x]$ is primitive. L is the length of the LFSR. $\alpha \in GF(2^L)$ is one root of $m_s(x)$. f_k denotes the unique maximum order term of a nonlinear k th-order function f applied to the LFSR's stages, $f_k = s_{n+t_0} s_{n+t_1} \cdots s_{n+t_{k-1}}$ where the symbols t_j ($j=0,1,\dots,k-1$) are integers verifying $0 \leq t_0 < t_1 < \cdots < t_{k-1} < 2^L - 1$. In this work only the contribution of f_k to the global linear complexity of the resulting sequence will be studied.

The *root presence test* for the product of k distinct phases of a PN-sequence can be stated as follows, [8]:

$\alpha^E \in GF(2^L)$ is a root of the minimal polynomial of the generated sequence if and only if

$$A_E = \begin{vmatrix} \alpha^{t_0 2^{e_0}} & \alpha^{t_1 2^{e_0}} & \cdots & \alpha^{t_{k-1} 2^{e_0}} \\ \alpha^{t_0 2^{e_1}} & \alpha^{t_1 2^{e_1}} & \cdots & \alpha^{t_{k-1} 2^{e_1}} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^{t_0 2^{e_{k-1}}} & \alpha^{t_1 2^{e_{k-1}}} & \cdots & \alpha^{t_{k-1} 2^{e_{k-1}}} \end{vmatrix} \neq 0$$

Here $\alpha^{t_j} \in GF(2^L)$ ($j=0,1,\dots,k-1$) correspond respectively to the k phases (s_{n+t_j}) of the PN-sequence. E , the representative element of the cyclotomic coset E , is a positive integer of the form $E = 2^{e_0} + 2^{e_1} + \cdots + 2^{e_{k-1}}$ with the e_i ($i=0,1,\dots,k-1$) all different running in the interval $[0, L)$. Under these conditions, α^E and its conjugate roots contribute to the global linear complexity of the nonlinearly filtered sequence. The value of this contribution is equal to the number of elements in such a cyclotomic coset.

The cyclotomic coset E is said to be *degenerate* if the corresponding determinant A_E equals zero. Otherwise the cyclotomic coset E will be *nondegenerate*.

Notice that every cyclotomic coset E can be easily associated with the radix-2 form of the integer E . This fact quite naturally suggests the introduction of binary strings of length L and Hamming weight k . Indeed, the cyclotomic coset E can be equivalently characterized by:

- (i) the integer E of the form $E = 2^{e_0} + 2^{e_1} + \cdots + 2^{e_{k-1}}$.
- (ii) an L -bit string whose 1's are placed at the positions $\{e_i\}_{i=0,1,\dots,k-1}$.
- (iii) the determinant A_E as defined before.
- (iv) the homogeneous linear system (2.1) associated with A_E ,

$$\begin{cases} 0 = d_0 \alpha^{t_0 2^{e_0}} + d_1 \alpha^{t_1 2^{e_0}} + \cdots + d_{k-1} \alpha^{t_{k-1} 2^{e_0}} \\ 0 = d_0 \alpha^{t_0 2^{e_1}} + d_1 \alpha^{t_1 2^{e_1}} + \cdots + d_{k-1} \alpha^{t_{k-1} 2^{e_1}} \\ \vdots \\ 0 = d_0 \alpha^{t_0 2^{e_{k-1}}} + d_1 \alpha^{t_1 2^{e_{k-1}}} + \cdots + d_{k-1} \alpha^{t_{k-1} 2^{e_{k-1}}} \end{cases} \quad (1)$$

where $d_j \in GF(2^L) \forall j$.

In the sequel these four characterizations will be used indistinctly. Regarding the use of the binary strings, some additional notation is necessary.

Let $E = 2^{e_0} + 2^{e_1} + \dots + 2^{e_{k-1}}$ and $F = 2^{f_0} + 2^{f_1} + \dots + 2^{f_{l-1}}$ be two L-bit strings of weight k and l respectively with $k < l$. $E \subset F$ means that $\{e_i\}_{i=0,1,\dots,k-1} \subset \{f_i\}_{i=0,1,\dots,l-1}$. That is, all the 1's in E are also in F .

For a set of L-bit strings $\{E_n\} = \{E_1, E_2, \dots, E_N\}$, $OR[\{E_n\}]$ denotes the L-bit string resulting from a bit wise OR among the L-bit strings of the set. Obviously, we have that $\forall n \in \{1, 2, \dots, N\}, E_n \subset OR[\{E_n\}]$.

Finally, we quote the following definitions and results related to the global linear complexity of a function with a unique term of maximum order, [1].

A cyclotomic coset is called a *fixed-distance coset* if it has an element E_d of the form $E_d = 2^{e_0} + 2^{e_1} + \dots + 2^{e_{k-1}}$, with $e_i \equiv d \cdot i \pmod{L} \forall i \in \{0, 1, \dots, k-1\}$ and d being a positive integer less than L such that $(d, L)=1$. Its name is due to the fixed distance d among the positions of the 1's in the L-bit string representation of E_d .

The 1 placed at the position e_j will be called the *jth-1* of the L-bit string associated with the coset E_d .

Theorem 1

f is a kth-order function if and only if all the fixed-distance cosets are non-degenerate.

Corollary 1

The global linear complexity Λ of the sequence produced by f is lower-bounded by $\Lambda \geq N_L \cdot L$, where $N_L = \frac{\Phi(L)}{2}$ ($\Phi(L)$ being the Euler function). Here N_L represents the number of fixed-distance cosets and L the number of elements in such cosets.

Corollary 2

If L is prime, then the global linear complexity Λ of the sequence generated by f is lower-bounded by $\Lambda \geq \binom{L}{2}$. Remark that these results, which constitute the starting point of the present work, are independent of the LFSR, the order of f and the particular form of f .

3 Theoretical Results

Considering a general function f defined as before, the present work is concerned with the next simple idea:

Not many degenerate cosets can exist simultaneously.

A proof of this statement can be outlined in three different steps. First, the N cosets of a specific set are supposed to be simultaneously degenerate. Then, it is proved that only m of these cosets (with $m < N$) can be simultaneously degenerate. Consequently, (N-m) cosets contribute to the global linear complexity of the resulting sequence.

This procedure can be expressed in a more formal way as follows. First of all, a new class of cosets is introduced.

Given a fixed-distance coset $E_d = 2^{e_0} + 2^{e_1} + \dots + 2^{e_{k-1}}$ and $j \in \{0, 1, \dots, k-1\}$, we will call *jth-quasi fixed-distance coset* (for short *jth-quasi f-d coset*) to any cyclotomic coset whose representative element F_d^j is of the form $F_d^j = 2^{f_0} + 2^{f_1} + \dots + 2^{f_{k-1}}$ such that $\{e_i\}_{i=0,1,\dots,k-1} \subset \{f_i\}_{i=0,1,\dots,k-1}$ $i \neq j$. That is, a jth-quasi f-d coset F_d^j is any cyclotomic coset whose L-bit string associated contains all the 1's of the L-bit string associated with E_d except for the jth-1.

$\{F_{d,n}^j\} = \{F_{d,1}^j, \dots, F_{d,N}^j\}$ is used to denote a set of jth-quasi f-d cosets.

Lemma 1

Let F_d^j be any jth-quasi f-d coset, then $A_{F_d^j}$ has at least a minor of order (k-1) (without the jth-row and an arbitrary ith-column) that does not equal zero:

$$\begin{vmatrix} \alpha^{t_0 2^{e_0}} & \cdot & \alpha^{t_{i-1} 2^{e_0}} & \alpha^{t_{i+1} 2^{e_0}} & \cdot & \alpha^{t_{k-1} 2^{e_0}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \alpha^{t_0 2^{e_{j-1}}} & \cdot & \alpha^{t_{i-1} 2^{e_{j-1}}} & \alpha^{t_{i+1} 2^{e_{j-1}}} & \cdot & \alpha^{t_{k-1} 2^{e_{j-1}}} \\ \alpha^{t_0 2^{e_{j+1}}} & \cdot & \alpha^{t_{i-1} 2^{e_{j+1}}} & \alpha^{t_{i+1} 2^{e_{j+1}}} & \cdot & \alpha^{t_{k-1} 2^{e_{j+1}}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \alpha^{t_0 2^{e_{k-1}}} & \cdot & \alpha^{t_{i-1} 2^{e_{k-1}}} & \alpha^{t_{i+1} 2^{e_{k-1}}} & \cdot & \alpha^{t_{k-1} 2^{e_{k-1}}} \end{vmatrix} \neq 0. \quad (2)$$

Proof The determinants $A_{F_d^j}$ and A_{E_d} differ exclusively in the jth-row. Expanding both determinants along the jth-row, we can write $A_{F_d^j}$ and A_{E_d} in terms of the k minors of order (k-1) of the form (3.1). The fact that $A_{E_d} \neq 0$ (see Theorem 1) completes the proof.

The following theorem is the theoretical basis of the LB-algorithm.

Theorem 2

Let E_d be any fixed-distance coset and $j \in \{0, 1, \dots, k-1\}$. If for some set of jth-quasi f-d cosets $\{F_{d,n}^j\}$ there exists at least a fixed-distance coset $E_{d'}$ such that $E_{d'} \subset OR[\{F_{d,n}^j\}]$, then the cosets of $\{F_{d,n}^j\}$ cannot be simultaneously degenerate.

Proof We proceed by contradiction. We assume that the cosets of $\{F_{d,n}^j\}$ are simultaneously degenerate. This simultaneous degeneration is equivalent to the existence of a set of homogeneous linear systems (associated with each determinant $A_{F_{d,n}^j}$) with nontrivial solutions. All these systems have (k-1) equations in common. Furthermore, due to Lemma 1, the solutions of each system are at the same time the joint solutions to all the systems, therefore the compatibility of the general system composed of all the different equations can be easily deduced. Finally, according to the starting hypothesis, the k equations associated with the determinant $A_{d'}$ are among the equations of the general system. This means that a compatible system has a non-compatible subsystem, which obviously is a contradiction.

The LB-algorithm that is presented in the next section realizes the previous results by means of the handling of L-bit strings.

4 The LB-Algorithm

In this section, the LB-algorithm which computes a lower bound on the global linear complexity is presented in detail. The LB-algorithm is based on the previous theorems and corollaries. For every set of N quasi f-d cosets, the algorithm determines:

- (a) the maximum number m of cosets which can be simultaneously degenerate.
- (b) the contribution to the global linear complexity of the $(N-m)$ remaining cosets which are nondegenerate.

The LB-algorithm converts the linear system (2.1) into an L-bit string according to the following simple rule: the presence of the i th-equation $0 = d_0\alpha^{t_02^{e_i}} + d_1\alpha^{t_12^{e_i}} + \dots + d_{k-1}\alpha^{t_{k-1}2^{e_i}}$ in the system implies a 1 in the L-bit string at the position indicated by e_i . Note that, due to the particular form of the linear system, squaring the equations of the system (2.1) is equivalent to a left cyclic rotation in the L-bit string associated (Fig. 1). This fact will be used widely throughout the algorithm.

4.1 Bit Wise Logic Operations

The LB-algorithm realizes basically three bit wise logic operations AND, OR and exclusive-OR (denoted by XOR). An interpretation of each operation is presented in the following.

Given two homogeneous linear systems and their corresponding binary strings, the AND operation between both strings gives rise to a new homogeneous linear system whose equations are common to both systems (Fig. 2 a)). In the algorithm the logic operation AND will be used to check the presence of a particular subsystem inside a general system.

The XOR operation of two L-bit strings associated with both linear systems of the form (2.1) gives rise to a new system whose equations belong exclusively to one of the previous linear systems (Fig. 2 b)). In the following algorithm the logic operation XOR is used to check if a particular coset has been previously studied.

Finally, the OR operation among several L-bit strings gives rise to a macrosystem which includes all the equations corresponding to the systems (Fig. 2 c)). Throughout the algorithm this logic operation is used as a fundamental tool to check the basic idea of this work: the simultaneous degeneration of the quasi f-d cosets.

It is clear that the LB-algorithm is based exclusively on the handling of L-bit strings instead of solving linear systems or computing determinants in a finite

field.

4.2 Notation

The following notation is used throughout the LB-algorithm.

FDC(i) ($i=1,2,\dots,N_L$) denotes the L-bit string corresponding to the i th-fixed-distance coset E_{d_i} .

Δ is a lower bound on the global linear complexity.

MASK(i,j) ($j=1,2,\dots,k-1$) denotes the L-bit string obtained from FDC(i) by replacing the j th-1 by a 0. Remark that MASK(i,0) is a shifted version of MASK(i,k-1).

$C(i,j)$ denotes a set of L-bit strings associated with the j th-quasi f-d cosets $\{F_{d_i,n}^j\}$. Any L-bit string in $C(i,j)$ previously considered must be eliminated. In order to detect them we operate every L-bit string in $C(i,j)$ as follows:

1. by means of AND operations with every FDC(i) ($i=1,2,\dots,N_L$) to discover the fixed distance cosets

2. by means of XOR operations with every previous MASK. Those cosets that produce a resulting string with a unique 1 must be eliminated from $C(i,j)$ as they have been already analysed in previous sets $\{F_{d_i,n}^j\}$.

m is a decreasing counter whose first value (denoted by M) is the number of L-bit strings in $C(i,j)$ after eliminations.

$a(n)$ ($n=1,2,\dots,\binom{M}{m}$) denotes each possible M-bit string of weight m .

VOR denotes the string resulting from an OR operation among those m cosets of $C(i,j)$ indicated by the positions of the 1's in $a(n)$.

VL is a binary variable whose value depends on the AND operation between VOR and each FDC(i).

4.3 Algorithm

The LB-algorithm INPUTS are L (LFSR's length) and k (order of the function) with $2 < k < L-2$, and its OUTPUT is the lower bound of the global linear complexity Δ .

Fig. 3 shows the LB-algorithm whose Steps 1 and 2 can be described as follows.

Step 1

Compute the N_L values of d .

Generate the FDC(i) ($i=1,2,\dots,N_L$).

Initialize the lower bound $\Delta = L \cdot N_L$.

Step 2

Generate MASK(i,j) ($i=1,2,\dots,N_L$; $j=1,2,\dots,k-1$).

Initialize the counter $m=L-k$.

Generate the set $C(i,j)$.

Realize the AND between every $FDC(l)$ ($l=1,2,\dots,N_L$) and every coset of $C(i,j)$. If any result equals $FDC(l)$, then the corresponding coset in $C(i,j)$ is eliminated and $m=m-1$.

Realize the XOR between every $MASK(o,p)$ ($o=1,2,\dots,i-1$, $p=1,2,\dots,k-1$; $o=i$, $p=1,2,\dots,j-1$) and every coset of $C(i,j)$. If any result has a unique 1, then the corresponding coset in $C(i,j)$ is eliminated and $m=m-1$.

4.4 Example

Fig. 4 shows the results obtained from the LB-algorithm for $L=11$ and $k=6$.

Since the LB-algorithm is independent of the specific function and minimal polynomial of the LFSR, the lower bound obtained is valid for any arbitrary nonlinear function with a unique term of maximum order 6 and for any maximal-length LFSR of length 11.

If we had used the root presence test to obtain the same result, we would have had to compute (for each function of order 6 and each maximal-length LFSR of length 11) at least 22 determinants of order 6 in $GF(2^{11})$. This would have implied more than a million arithmetic operations in a finite field, [4]. According to the present algorithm, the numerical result obtained is independent of the function and the maximal-length LFSR.

4.5 Discussion

The main facts concerning the performance of the algorithm are summarized in this section.

The LB-algorithm is divided into two stages. The first stage includes the generation and ‘debugger’ of the cosets to be analysed. The second stage is concerned with the simultaneous degenerations of the different sets of cosets. In the second stage a ‘sweep’ of some sets of cosets is carried out, which permits their use later on the algorithm.

Regarding the required memory, note that only the L -bit strings $MASK(i,j)$ (but not the cosets $C(i,j)$) have to be stored. This means keeping one out of $(L-k)$ cosets analysed.

In order to handle the cosets of $C(i,j)$, the more suitable structure of information is a list. This structure seems also adequate to select, through the codification $a(n)$, the cosets involved in each OR operation. On the other hand, in order to generate the successive strings $a(n)$, backtracking can be used.

It can also be determined that the LB-algorithm has a maximum computational complexity of order $O(2^{L-k})$, where L denotes the length of the LFSR and k is the order of the function. In order to estimate this value, it has been assumed the ‘worst possible case’, which involves a number of logic operations given by $N_L(k-1)[\binom{M}{M} + \binom{M}{M-1} + \dots + \binom{M}{2}] = N_L(k-1)(2^M - M) \leq N_L(k-1)2^{L-k}$. However, from the experimental results it can be deduced that the running time of the LB-algorithm depends on the real number of bit wise operations

among the different L-bit strings, which is much less. As an illustrative example we can say that for L=53 and k=27 the number of logic operations is only $N_{53}(27-1)[\binom{25}{25} + \binom{25}{24} + \binom{25}{23} + \binom{25}{22} + \binom{25}{21}] \leq N_{53} \cdot 26 \cdot 2^{26}$.

Furthermore the following three considerations must be taken into account. First, for each pair of values (L,k), the LB-algorithm has to be used only once. Second, it will be used only with relatively small inputs. And third, a high bound obtained for specific values of L and k will encourage the designer of running-key generators to use nonlinear filter with a unique term of maximum order k applied to any maximal-length LFSR of length L.

The LB-algorithm has been implemented on a DEC work-station and several experiments over values of L primes have been carried out to evaluate it. The effect of this choice is twofold. On the one hand, it simplifies the computation of the N_L values of d in Step 1, and on the other hand, the more fixed-distance cosets there are the higher bounds the algorithm computes.

The following table shows some experimental results.

L	11	17	23	29	37	43	47	53
k	6	9	12	15	19	22	24	27
Bound	242	3128	8349	22330	47952	75852	99405	143206

Table 1: Lower bounds on the global linear complexity

According to the values shown, the LB-algorithm is believed to be quite efficient to lowerbound the global linear complexity of the filtered sequences. The growth of the bound observed can be approximated by the curve of Fig 5, which has been obtained through regression analysis for the linear model. This approximation let us estimate a bound above 500000 for L=89.

In conclusion, the main result deduced from the LB-algorithm is reliability for the nonlinear filter. Thanks to it a designer of nonlinear filter generators could carry out the following steps:

- 1.- Find values of L and k that produce a high lower bound,
- 2.- Choose any nonlinear function of a smaller order than k,
- 3.- Add it to any kth-order product and
- 4.- Apply the resulting nonlinear function to any maximal-length LFSR of length L.

In this way the designer would obtain a sequence with a guaranteed large global linear complexity.

5 Conclusions

Our research has highlighted the problem of the global linear complexity of the nonlinear filter generators. In addition, a new algorithm, the so-called LB-algorithm, to lowerbound the global linear complexity has been presented.

This proposal differs from existing schemes in different aspects. Firstly, unlike the well-known Berlekamp-Massey's algorithm [6], we do not consider

the digits of the output sequence but the characteristics of the nonlinear filter. Secondly, the proposed algorithm indeed does not require any condition on the LFSR's stages involved, as do [5] and [7]. Therefore the obtained bounds are valid for any nonlinear function with a unique term of maximum order. Finally, this work is based on the handling of L-bit strings instead of computing determinants in a finite field (Rueppel's method, [8]), which seems to be much more adequate for software simulation and/or hardware implementation.

Large lower bounds for the global linear complexity have been obtained from the LB-algorithm without imposing any restriction on the function or the polynomial. This fact ensures the reliability of the nonlinear state-filter generators for cryptographic application.

This investigation has left as open problem the study of the remaining cosets that the LB-algorithm does not analyse.

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